Answers are found at the bottom of the worksheet on a seperate page.

Question 1: Consider the following function: $f : \mathbb{Z} \to \mathbb{N} \cup \{0\}, f(x) = x^2$. Note: Here, we are telling you how x gets mapped. Another way to say this is $x \mapsto x^2$, or, x gets mapped to x^2.

Determine the following about the function above: Whether this is a function, Domain, Codomain, Range, Whether the function is injective, surjective, bijective or none. You don't have to prove your results.

Question 2: Consider the following function: $f : \mathbb{R} \to \mathbb{Q}, f(x) = x$, or, $x \mapsto x$.

Determine the following about the function above: Whether this is a function, Domain, Codomain, Range, Whether the function is injective, surjective, bijective or none. You don't have to prove your results.

Question 3: What is the set $A = (\mathbb{Z} \setminus \mathbb{N}) \cup \{\frac{1}{2}\}$?

Question 4: Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^3 - 1$. Find the inverse of this function, as you would in a calculus class.

While we didn't cover this in the videos, this is a warmup to our future exposure to inverses.

Remark: Let's say $A = \{1, 2, 3, 4, 5\}$, and $f : A \to A$. There are two interesting questions we can ask.

a: How many distinct functions are there? As in, what are all the ways we can map elements in A to itself?

b: How many of these functions are bijections? Or, maps to itself uniquely.

The answer to a: 5^5. 1 can map to 5 different elements, so 5 choices. 2 can map to 5 elements independent of 1, so $5 \cdot 5 = 25$ choices. Then, 3 introduces 5 more choices, so 5^3 until the fifth element where we end with 5^5 choices.

The answer to b: $5! = 120 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. The first element can map anywhere, so 5 choices. But the second element can't map to where the first element mapped, so $5 \cdot 4 = 20$ choices. Then, the third element can't map to where the first element or second element was mapped, so $20 \cdot 3 = 60$ choice, until we end with 120 choices.

This is a question that you might encounter in a field of mathematics called combinatorics, or the math of counting things. While this remark might be hard to understand, I hope it was interesting!

Answer 1:

Function: Yes

Domain: $\mathbb{Z}$

Codomain: $\mathbb{N} \cup \{0\} = \{0, 1, 2, 3, \ldots\}$

Range: $\{0, 1, 4, 9, 16, 25, \ldots\}$, or, the square numbers.

f is neither injective or surjective. Both 2 and -2 map to 4 so it fails to be injective, and 3 is not reached under the function.

Answer 2:

f isn't a function (don't worry, I made the same mistake when coming up with the question), since π doesn't map anywhere. So it doesn't make sense to ask about its properties.

If you want, change f so that $f : \mathbb{Q} \to \mathbb{R}$. Now, what properties does f have?

Domain: $\mathbb{Q}$

Codomain: $\mathbb{R}$

Range: $\mathbb{Q}$

f is injective but not surjective, as π isn't mapped to. We won't prove the injective claim.

Answer 3: $A = \{\ldots, -3, -2, -1, 0, \frac{1}{2}\}$.

Answer 4: The way I find inverses is by changing x and y, and then resolving for y. So we get:

$$\begin{aligned} x &= y^3 - 1 \\ y^3 &= x + 1 \\ y &= \sqrt[3]{x+1} \end{aligned}$$

Thus, $f^{-1} = \sqrt[3]{x+1}$.