An Introduction to Mathematical Proofs

Functions

Who? Fahad Hossaini

When? Whenever You Watch

I/O

At a basic level, we think of functions as input/output machines. However, how can we formalize this? What properties do functions have, and why are they important?

We'll explore this using a diagram! It's Manim Time!

Let's Do A Quick Recap!

A function is a map from set A to set B that follows two rules.

1: An element in A gets mapped to only one value in B

2: Every element in A gets mapped to elements in B

We denote this as: $f : A \rightarrow B$

Definitions

Let $f : A \to B$ be our function. Then,

Definition Domain: The Input Set. In this case, A.

Definition Codomain: The Output Set, or, the set of all possible outputs. In this case, B.

Definition Range: The Reachable Set, or, the set of all achievable outputs. Elements in this set get mapped to under f. $\text{Range}(f) \subseteq B$.

Definition Injective: Each input is mapped to a unique output. i.e Every element in the range is mapped to uniquely.

Definition Surjective: $\text{Range}(f) = B$. Or, all possible outputs are achievable.

Definition Bijective: Injective and Surjective. A bijective function is invertible, resulting in $f^{-1} : B \to A$.