Answers are found at the bottom of the worksheet on a seperate page.

Question 1: Is $A = \{3, 7, 8.5, \{-2, 1\}, -2, 1\}$ a set?

Question 2: Is $B = [0, 1) \cup (-1, 2]$ a set?

Question 3: Is $C = \{1, 1.1, 1.11, 1.111, 1, 1111\}$ a set? Hint: One of those commas isn't actually a comma...

Question 4: Informally argue that $A \cap B \subseteq A \cup B$. We'll formally prove this in the answers since it wasn't covered in the videos.

Question 5: How many unique subsets of $A = \{1, 2, 3\}$ are there? List them all.

Question 6: List all the elements in the following set:

$A = \{(x, y) : 0 \leq x < 2, y$ is prime and $0 < y < 10,$ x and y are non-decimal numbers$\}$

Question 7: Show that the set $A = \{x + 1 : x \text{ is even}\}$ is equal to $B = \{x : x \text{ is odd}\}$, where x are non-decimal numbers.

Question 8: What is $\{1, 2, 3, 4\} \setminus (0, 5]$? Write this in terms of open intervals and set operations.

Question 9: If $A = \{2, 5, 9\}$, list all the elements in $A \times A$.

Remark: At the end of the answers, we'll show that $A \cap B^C = A \setminus B$.

Answer 1: Yes, A is a set. Remember, order doesn't matter and it contains no duplicate elements as $\{-2, 1\} \neq -2 \neq 1$.

Answer 2: Yes, when we union sets, even if they overlap, they contain no duplicates. Set operations always result in sets.

Answer 3: No, 1 is listed twice. Sorry about this one...

Answer 4: Intuitively, this makes sense, the elements in both sets must be in one of the sets. So we'll prove this:

Proof. Let A and B be sets. If $A \cap B = \varnothing$, then this statement is vacuously true as the null set is a subset of every set.

Let $x \in A \cap B$ be an arbitrary element. By definition of intersection we have: $x \in \{x : x \in A \text{ and } x \in B\}$.

Thus, x is in A and B. But then clearly, x is in A. So, $x \in \{x : x \in A \text{ or } x \in B\} = A \cup B$.

$\square$

Note, the last paragraph is true because of the word 'or' and knowing what it means. Thus, we can also show that $A \subseteq A \cup B$.

Answer 5: There are 8 unique subsets of A. In general, if n is the number of elements in a finite set A, then we say that the number of unique subsets of is 2^n. This should intuitively make sense and can be proven later with induction.

The unique subsets are: $\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$.

Answer 6: $A = \{(0, 2), (0, 3), (0, 5), (0, 7), (1, 2), (1, 3), (1, 5), (1, 7)\}$

Note: 1 isn't prime. Also, these are pairs of elements, as dictated by set builder notation.

Answer 7: This is a double subset inclusion proof.

Proof. First, we'll show that $A \subseteq B$.

Let $y \in A$. Then, $y = x + 1$ where x is even. So, $x = 2k$.

Then, $y = 2k + 1$ is odd, so $y \in B$. Thus, every element of A is in B since an arbitrary element, or an element of the most general form in A is also in B.

So $A \subseteq B$. Now, we'll show that $B \subseteq A$.

Let $y \in B$. Then $y = 2k + 1$ as y is odd.

Let $x = 2k$. Then, x is even.

So, $y = x + 1 \in A$. Thus, every element of B is in A since an arbitrary element, or an element of the most general form in B is also in A.

Since $B \subseteq A$ and $A \subseteq B$, we have by definition that $A = B$. $\square$

Answer 8: The set is: $(0, 1) \cup (1, 2) \cup (2, 3) \cup (3, 4) \cup (4, 5]$

Answer 9: $A \times A = \{(2, 2), (2, 5), (2, 8), (5, 2), (5, 5), (5, 8), (8, 2), (8, 5), (8, 8)\}$

Remark We'll prove that $A \cap B^C = A \setminus B$.

Proof. This is a double subset inclusion. Let A and B be arbitrary sets. We'll first show that $A \cap B^C \subseteq A \setminus B$. If $A \cap B^C = \varnothing$, we are done. Otherwise, let $x \in A \cap B^C$.

By definition, we have:

$$
\begin{aligned}
x &\in A \cap B^C \\
&\in \{x : x \in A \text{ and } x \in B^C\} \\
&\in \{x : x \in A\} \text{ and } \{x \in B^C\} \\
&\in A \text{ and not } \{x \in B\} \\
&\in A \text{ and not } B \\
&\in A \setminus B
\end{aligned}
$$

Thus, $x \in A \setminus B$. The reverse subset inclusion is the same but with the steps reversed. Thus, $A \cap B^C = A \setminus B$. $\square$

So a few things to say about this proof: We deconstruct set builder and definitions to achieve this result. But this deconstruction happens in logically sound ways. For example, $x \in B^C$ is the same as saying $x \notin B$. Or, A and not B is the same as $A \setminus B$.

Keep in mind, this means understanding how each set operations and definitions work. However, in practice, we won't need more than simple set operations and simple understanding of set builder notation.