This worksheet will be a little different from the rest. I didn't cover sets properly in the video since it's not super necessary for the rest of the course. As long as you know the concepts, you're fine. Regardless, that one's my bad.

There will be exposition/teaching after question 5. If you prefer, you can do the questions directly, but you might not be able to do them without the exposition.

As always, answers are found at the bottom of the worksheet on a seperate page.

Question 1: Is $A = \{3, 7, 8.5, \{-2, 1\}, -2, 1\}$ a set?

For this question 1,2 and 3, set is defined as: An unordered collection of distinct objects.

Question 2: Is $B = [0, 1) \cup (-1, 2]$ a set?

Question 3: Is $C = \{1, 1.1, 1.11, 1.111, 1, 1111\}$ a set?

Hint: One of those commas isn't actually a comma...

Question 4: How many unique subsets of $A = \{1, 2, 3\}$ are there? List them all.

Question 5: List all the elements in the following set:

$A = \{(x, y) : 0 \leq x < 2, y$ is prime and $0 < y < 10,$ x and y are non-decimal numbers$\}$

Question 6: If $A = \{2, 5, 9\}$, list all the elements in $A \times A$.

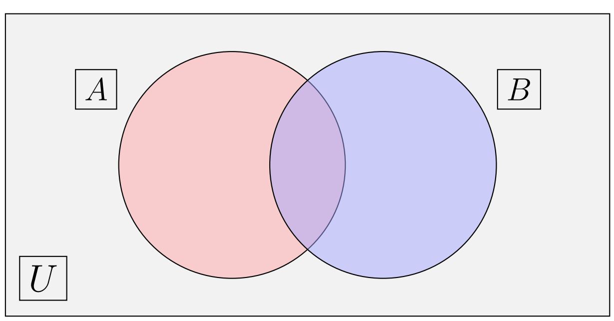
Teaching time!

They're two things I want to clarify here. First, set operations.

We covered the main five set operations. Union, Intersection, Difference, Complement and Cartesian Product. However, there's a way to visualize the first four operations.

Imagine that the following box is U, or the universal set. We'll color it partially gray.

Now, we'll add two sets inside U. Namely, A and B. Both A and B will be circles. Everything in red will be in A while everything in blue is in B.



I want to make the following fact clear: We aren't explicitly saying what elements are in A or B.

Rather, we think of elements as being some point in this box somewhere. If the point is in the red section, it's an element of A. If the point is in the blue section, it's an elements of B.

Now, we can think of our set operations from a visual perspective.

First, union. Union is any point in either A or B denoted as $A \cup B$. In the diagram, this is anywhere in the venn diagram (or any colored portion, where colored means red, purple and blue).

Outside the venn diagram, or in the empty space where a point is neither in A or B isn't in the union. We could also call this space, which is the grey section, the complement of the union, or, $(A \cup B)^C$

Second, intersection. Intersection is any point in both A or B denoted as $A \cap B$. This is the purple area, or where both circles intersect.

Third, Difference. We'll consider $A \setminus B$, or the elements in A that's not in B. This includes only elements in red, *not purple*.

Likewise, if we consider $B \setminus A$, this is all the elements in B that's not in A. This includes only elements in blue, *not purple*.

Finally, since Cartesian Product can't be visualized this way (maybe you can see why), we'll look at complement. Complement is everything *not* in a set.

For example, we said before that $(A \cup B)^C$ is the grey area in the box, or anything not in color.

Let's look at A^C, or the elements not in A. This consists of the grey and blue area. Purple is in A, so it can't be in the complement.

Lastly, B^C is the area in grey and red.

Okay, with set operations done with, we'll do a subset inclusion proof.

Prove that $A \cap B \subseteq A \cup B$.

Intuitively, this makes sense, the elements in purple are in red, purple and blue combined. We'll prove this:

Proof. Let A and B be sets. If $A \cap B = \varnothing$, then this statement is vacuously true as the null set is a subset of every set.

Let $x \in A \cap B$ be an arbitrary element. By definition of intersection we have: $x \in \{x : x \in A \text{ and } x \in B\}$.

Thus, x is in A and B. But then clearly, x is in A. So, $x \in \{x : x \in A \text{ or } x \in B\} = A \cup B$.

$\square$

Note, the last paragraph is true because of the word 'or' and knowing what it means. Thus, we can also show that $A \subseteq A \cup B$ with no difficulty.

The key here is leveraging english.

Here's another way to proceed with subset proofs. We'll prove that $A \cap B^C = A \setminus B$.

Proof. This is a double subset inclusion. Let A and B be arbitrary sets. We'll first show that $A \cap B^C \subseteq A \setminus B$. If $A \cap B^C = \varnothing$, we are done. Otherwise, let $x \in A \cap B^C$.

By definition, we have:

$$
\begin{aligned}
x &\in A \cap B^C \\
&\in \{x : x \in A \text{ and } x \in B^C\} \\
&\in \{x : x \in A\} \text{ and } \{x \in B^C\} \\
&\in A \text{ and not } \{x \in B\} \\
&\in A \text{ and not } B \\
&\in A \setminus B
\end{aligned}
$$

Thus, $x \in A \setminus B$. The reverse subset inclusion is the same but with the steps reversed. Thus, $A \cap B^C = A \setminus B$. $\square$

So a few things to say about this proof: We deconstruct set builder and definitions to achieve this result. But this deconstruction happens in logically sound ways. For example, $x \in B^C$ is the same as saying $x \notin B$. Or, A and not B is the same as $A \setminus B$.

Keep in mind, this means understanding how each set operations and definitions work. However, in practice, we won't need more than simple set operations and simple understanding of set builder notation.

Okay, now we can proceed with the questions!

Question 7: Show that the set $A = \{x + 1 : x \text{ is even}\}$ is equal to $B = \{x : x \text{ is odd}\}$, where x are non-decimal numbers.

Question 8: What is $\{1, 2, 3, 4\} \setminus (0, 5]$? Write this in terms of open intervals and set operations.

Question 9: Draw a picture for the following sets:

1. $\mathbb{R} \times [0, 1]$
2. $[0, 1] \times \mathbb{R}$
3. $([1, 2] \cup [3, 4]) \times ([-1, 0] \cup [2, 3])$

Question 10: Let A, B, C be sets. Provide a counterexample for the following false set identity. (Draw a Venn diagram first with three circles to get an idea of why the question is false.)

$A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$

Question 11: Let A, B, C be sets. Provide a counterexample for the following false set identity. (Draw a Venn diagram first with three circles to get an idea of why the question is false.)

$(A \cap B) \setminus C = A \cap C^c$.

Question 12: Prove the following set identities where A, B, C are sets.

$A \cap B \cap C = (A \cap B) \cap (B \cap C) \cap (A \cap C)$

Question 13: Prove the following set identities where A, B are sets.

$A \subseteq B$ if and only if $A \setminus B = \emptyset$.

Remark: Set theory is surprisingly complex. We tried to derive all of mathematics given a few rules or axioms that we take for granted, but that's impossible. Yes, math is technically incomplete.

Anyways, we'll talk more about sets later on in the series.

Answer 1: Yes, A is a set. Remember, order doesn't matter and it contains no duplicate elements as $\{-2, 1\} \neq -2 \neq 1$.

Answer 2: Yes, when we union sets, even if they overlap, they contain no duplicates. Set operations always result in sets.

Answer 3: No, 1 is listed twice. Sorry about this one...

Answer 4: There are 8 unique subsets of A. In general, if n is the number of elements in a finite set A, then we say that the number of unique subsets of is 2^n. This should intuitively make sense and can be proven later with induction.

The unique subsets are: $\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$.

Answer 5: $A = \{(0, 2), (0, 3), (0, 5), (0, 7), (1, 2), (1, 3), (1, 5), (1, 7)\}$

Note: 1 isn't prime. Also, these are pairs of elements, as dictated by set builder notation.

Answer 6: $A \times A = \{(2, 2), (2, 5), (2, 8), (5, 2), (5, 5), (5, 8), (8, 2), (8, 5), (8, 8)\}$

Answer 7: This is a double subset inclusion proof.

Proof. First, we'll show that $A \subseteq B$.

Let $y \in A$. Then, $y = x + 1$ where x is even. So, $x = 2k$.

Then, $y = 2k + 1$ is odd, so $y \in B$. Thus, every element of A is in B since an arbitrary element, or an element of the most general form in A is also in B.

So $A \subseteq B$. Now, we'll show that $B \subseteq A$.

Let $y \in B$. Then $y = 2k + 1$ as y is odd.

Let $x = 2k$. Then, x is even.

So, $y = x + 1 \in A$. Thus, every element of B is in A since an arbitrary element, or an element of the most general form in B is also in A.

Since $B \subseteq A$ and $A \subseteq B$, we have by definition that $A = B$. $\square$

Answer 8: The set is: $(0, 1) \cup (1, 2) \cup (2, 3) \cup (3, 4) \cup (4, 5]$

Answer 9:

Answer 10:

Answer 11:

Answer 12:

Answer 13: